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Equivalence principle, gravitational collapse, and the classical particle problem

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Abstract. We compare various modifications of general relativity theory (GRT) from the viewpoint of the equivalence principle. In GRT, gravitational collapse and the classical particle problem are closely connected with Einstein's version of the strong principle of equivalence. It is argued that theories which violate that principle or start from its 'telescopic' formulation might avoid collapse. On the other hand, one should invoke Einstein's equations with fourth-order corrections in order to solve the classical particle problem.

There exist in GRT two fascinating problems closely connected with the strong principle of equivalence, namely the gravitational collapse and the classical particle problem. Gravitational collapse leads inevitably to a singularity (more precisely, to a dynamic singularity) and all models of classical particles are necessarily described by (potential-theoretical) singularities.

To express more clearly the link between the equivalence principle and the singularities, we first consider a plausible argument for the occurrence of gravitational collapse in the Newtonian language. The strong principle of equivalence may then be written as

$$m_{\mathrm{I}} = m_{\mathrm{A}} = m_{\mathrm{P}} \tag{1}$$

where m_{I} is the inertial, m_{A} the active gravitational, and m_{P} the passive gravitational mass.

It is our object to consider a simple model of a gravitating mass. If its inertial mass is increased by additional baryons, then the active gravitational mass too grows automatically by reason of equation (1)—up to the point in time when the inner pressures are unable to stabilise self-gravitation. This situation does not cogently lead, in Newtonian theory, to dynamic singularities, because the fundamental laws of this theory do not furnish any restrictions with regard to the equations of state. In contrast, special relativity theory implies conditions on these equations resulting from the principles of causality and locality, and therefore the relativistic generalisation of equation (1) simultaneously imposes conditions for density and pressure—such that not all sufficiently large distributions of mass can be in a stable state. By the same token, the self-forces affect the 'collapse'' of the gravitational elementary field sources.

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The most general-relativistic version of equation (1) (strong principle of equivalence) demands: (i) that the components of the metric $g_{\mu\nu}$ are the only functions describing gravitation; (ii) that the gravitational equations derive from a variational principle. If we confine ourselves to dealing with field equations of second order, this equivalence principle admits Einstein's field equations alone, namely,

$$R^{\nu}_{\mu} - \tfrac{1}{2}\delta^{\nu}_{\mu}R = (-8\pi G/c^4)T^{\nu}_{\mu}. \tag{2}$$

As a consequence of equation (2), the inertial mass (Tolman 1934)

$$m_{\rm I} = \iiint \sqrt{-g}(T^4_4 + t^4_4)\,{\rm d}x^1\,{\rm d}x^2\,{\rm d}x^3 \tag{3}$$

(t^{ν}_{μ} is Einstein's pseudotensor) of isolated quasistatic objects is equivalent to the active gravitational mass $m_{\rm A}$, determined by the third law of Kepler. Consideration of $m_{\rm I} = m_{\rm P}$, following† from

$$T^{\nu}_{\mu\ ,\nu} = 0 \tag{4}$$

shows that GRT indeed generalises equation (1).

In order to avoid gravitational collapse one should attempt to work with second-order derivative equations which differ from Einstein's equations, the reason being that, as the plausibility argument mentioned above demonstrated, one has to surrender the strong principle of equivalence. But field equations of fourth order stemming from the variational principle

$$\delta \int \sqrt{-g}[l^2(\alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2) + R]{\rm d}^4x = 0 \tag{5}$$

are hardly suitable, because they are again a realisation of the strong principle of equivalence. A theory invoking equation (5), is, however, interesting with regard to the particle problem.

Let us consider some modifications of GRT to attain a gravito–dynamics where the quasi-Newtonian attraction tends to zero in regions of extremely large masses and densities, or even goes over into repulsive forces. The necessary modifications of the Poisson equation

$$\Delta\phi = 4\pi G\rho \tag{6}$$

and of Einstein's equations (2), respectively, may then be realised by changing the coupling between gravitation and matter.

First, it is possible to replace the right-hand side of equation (6) by the term $\rho\phi/c^2$. A general-relativistic formulation of such a potential-like coupling of gravitation and sources was accomplished with the aid of tetrad theories of gravitation (Treder 1971). In those theories, the gravitational potential is described by the tetrads $h^{\rm A}_{\mu}$ connected with the metric $g_{\mu\nu}$ via

$$h^{\rm A}_{\mu}h^{\rm B}_{\nu}\eta_{\rm AB} = g_{\mu\nu}. \tag{7}$$

The relativistic correlate of $\rho\phi/c^2$ now reads:

$$(h^{\rm A}_{\mu} - \delta^{\rm A}_{\mu})T^{\mu}_{\nu}. \tag{8}$$

† The mass $m_{\rm I}$ dealt with here is indeed identical with the mass $m_{\rm I}$ defined by equation (3), because equation (4) is a consequence of the field equations (2) and thus of the fact that equation (2) derives from a variational principle.

From equation (8) it is evident that the pure vacuum part, appearing on the right-hand side of the gravitational equations, will be entirely different from the Einstein tensor. In Treder's tetrad theory, this part is constructed in such a manner that the $g_{\mu\nu}$ in the linear approximation of the gravitational equations result in the Newtonian–Einstein vacuum.

Second, it is possible to modify the coupling without changing the Einstein vacuum. To this end one has to resort to a new universal constant L of the dimension of length, so that the source term may be corrected by terms proportional to differential operators acting upon the gravitational field. The Poisson equation must then be changed into

$$\Delta\phi = 4\pi G[\rho + \beta(L^2/c^2)(\Delta\phi)\rho]. \tag{9}$$

Such an *ansatz* may be formulated relativistically by adding crossing terms to Einstein's equations which are bilinear in the curvature tensor $R_{\mu\nu\alpha\beta}$ and the matter tensor $T_{\mu\nu}$, e.g., $L^2R_{\mu\alpha\beta\nu}T^{\alpha\beta}$, $L^2g_{\mu\nu}R_{\alpha\beta}T^{\alpha\beta}$:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = -\kappa(T_{\mu\nu} + L^2\theta_{\mu\nu}),$$

where

$$\theta_{\mu\nu} = \alpha R_{\mu\alpha\beta\nu}T^{\alpha\beta} + \beta g_{\mu\nu}R_{\alpha\beta}T^{\alpha\beta} + \dots \tag{10}$$

and α, β are dimensionless numbers (Liebscher *et al* 1977).

Both modifications differ in that the first becomes physically significant for strong potentials $|\phi|(\phi \approx c^2)$, and that the second balances Newtonian attraction for large densities $(\beta L^4G\rho/c^2 \approx 1)$. Nevertheless, both modifications are analogous in that they violate the strong principle of equivalence, though in a different manner. The first breaks the equality $m_A = m_I$, because it satisfies equation (4), but uses more functions than the ten $g_{\mu\nu}$ to describe gravitational fields. The second breaks the equality $m_I = m_P$, since from equation (10) it follows that $T_{\mu;\nu}^{\nu} \neq 0$.

The modifications (8) and (10) are chosen such that in the vacuum those theories lead to either the Laplace equation $\Delta\phi = 0$ or to the Einstein equations $R_{\mu\nu} = 0$. The modified gravitational potential in the interior of matter distribution manifests itself in the exterior of masses only by the conditions which result from joining vacuum and matter solutions. Indeed, the right-hand side of equation (9) may be developed in a series beginning with the terms $4\pi G(\rho + 4\pi GL^2\beta\rho^2/c^2)$ so that, if one invokes Green's theorem, equation (9) yields for the effective gravitational mass

$$GM = G\int\left(1 + \frac{4\pi GL^2\beta\rho}{c^2}\right)\rho\, d^3x. \tag{11}$$

For $\beta < 0$, $GM < G\int\rho\, d^3x$.

In order to solve the classical particle problem, one has to resort to equations modifying strongly the Einstein vacuum $R_{\mu\nu} = 0$ at short distances. Only if one employs such equations can one hope to find regular vacuum solutions (solitons) representing field models of particles. Of course, there is the restriction that for large distances these equations give the Newton–Einstein vacuum as an approximate solution. These considerations suggest the unitary field equations stemming from the Lagrangian (Treder 1977, von Borzeskowski *et al* 1978)

$$\mathscr{L} = \sqrt{-g}[R + L^2(\alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2) + \text{higher-derivative invariants}] \tag{12}$$

containing equation (5) as a particular case. Those equations also furnish other vacuum solutions as well as the special Einstein spaces $R_{\mu\nu}=0$. Accordingly the elementary solution of the linearised version of this theory

$$\Delta\phi - L^2\Delta^2\phi = 0 \tag{13}$$

is given by the regular Green function

$$\phi \propto (1/r)(1-\exp(-r/L)).$$

The modified equations following from the Lagrangian (12), written symbolically as

$$A_{\mu\nu} \equiv L^2 H_{\mu\nu} + (R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R) = 0 \tag{14}$$

possibly eliminate potential-theoretical singularities from the gravitational theory. Such a possibility is plausibly supported by the fact that the quantised equations are renormalisable (Utiyama and DeWitt 1963, Deser 1976, Stelle 1977). If one does not choose the constant L appearing in equation (12) as a new fundamental constant, but as Planck's length $(hG/c^3)^{1/2}$, then this theory may be interpreted as some phenomenological consequence of quantised GRT (von Borzeszkowski *et al* 1978).

Since equation (14) follows from a variational principle, the differential identity $A_{\mu;\nu}^{\ \nu}=0$ is satisfied. Accordingly, for matter coupled to gravitation, the equation $T_{\mu;\nu}^{\ \nu}=0$ is also satisfied, in contrast to the theory given by equation (10). This results from the validity of the strong principle of equivalence.

Finally, in this connection a different approach to solve the problem of gravitational collapse ought to be mentioned. As was already noted above, the modification in tetrad theories becomes physically significant for $\phi \approx c^2$, i.e. for

$$GM/c^2 \approx G\rho r^3/c^2 \geqslant r. \tag{15}$$

Since the condition (15) states that the modification of the gravitational potential becomes effective if the gravitational radius GM/c^2 is greater than the geometric dimension of the object in question, the tetrad theories contain a dependence upon the global properties of the system. This theory resembles in this respect the situation one encounters in the so-called Riemann–Mach mechanics proposed by Treder to formulate the Mach–Einstein doctrine (Treder 1972).

A mechanics of this kind attributes the inertial mass

$$m^* = m(1+2\beta|\phi|/c^2) \tag{16}$$

to each particle of a distribution of mass possessing the average gravitational potential ϕ (β is a numerical constant). Gravitational collapse is therefore prevented, because the maximal velocity of particles produced by gravitation can never exceed the value $(c/\beta^{+1/2})$ (for $\beta>1$). This effect results directly from Mach's principle expressing an induction of inertia by the gravitational potential, in the sense that an increase of the gravitational potential is always accompanied by an increase of inertia. In the interior of a system with the Newtonian total mass $M=Nm$ (N = particle number), gravitation has an effect which corresponds, according to Newton's second law, to a mass

$$M^* = \frac{M}{1+2\beta|\phi|/c^2}. \tag{17}$$

There exist therefore—due to the equilibrium between gravitation and induced inertia—stable configurations of astrophysical objects. Looked at from the exterior, the system acts upon other objects in such a manner that all its masses (gravitational and inertial) are identical. Hence the Mach–Einstein doctrine is a cosmological realisation of the strong principle of equivalence.

To sum up, we should emphasise the following fact: In order to solve the problem of gravitational collapse without surrendering Einstein's strong principle of equivalence, the cosmological approach is the most attractive one. But to solve the particle problem by having recourse to this principle, it seems most promising to invoke the Einstein equations with fourth-order derivative corrections. All other modifications of GRT prove to be less satisfactory, since they violate the strong principle of equivalence.

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